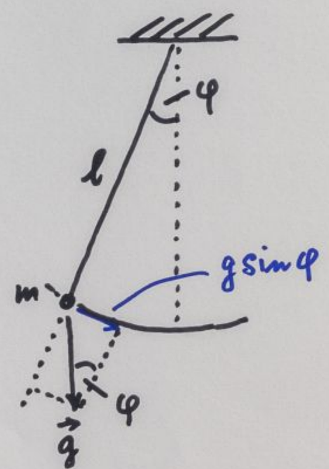
Kleinwinkel - Näherung und Eulersche Formel

a) $f(x) = \sin x$, McLaurin - Reihe

$$f(x) = \sin x , \quad f(0) = 0$$
$$f'(x) = \cos x , \quad f'(0) = 1$$
$$f''(x) = -\sin x , \quad f''(0) = 0$$
$$f'''(x) = -\cos x , \quad f'''(0) = -1$$

$$\underline{\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - + \dots} \quad , \text{ ungerade}$$

Beispiel: Fadenpendel, Kreispendel



Bewegungsgleichung:

$$l m \ddot{\varphi} = - m g \sin \varphi$$

$$l \ddot{\varphi} + g \sin \varphi = 0$$

Kleinwinkelnäherung:

$$\sin \varphi \approx \varphi$$

$$\ddot{\varphi} + \frac{g}{l} \varphi = 0$$

$\varphi = 5°$: $\hat{\varphi} = 0{,}08727$, $\sin \varphi = 0{,}08716$

$\varphi = 10°$: $\hat{\varphi} = 0{,}17453$, $\sin \varphi = 0{,}17365$

b) $f(x) = \cos x$, $f(0) = 1$

$$f'(x) = -\sin x , \quad f'(0) = 0$$
$$f''(x) = -\cos x , \quad f''(0) = -1$$
$$f'''(x) = \sin x , \quad f'''(0) = 0$$

$$\underline{\cos x = 1 - \frac{1}{2} x^2 + \frac{x^4}{24} - + \dots} \quad , \text{ gerade}$$

c) $f(x) = e^x$, $f(0) = 1$

$$f^{(n)}(x) = e^x , \quad f^{(n)}(0) = 1$$

$$\underline{e^x = 1 + x + \frac{1}{2} x^2 + \frac{1}{6} x^3 + \dots} = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

d) Eulersche Formel

$$x \to ix , \quad i^2 = -1 , \quad i^3 = -i , \dots$$

$$e^{ix} = 1 + ix - \frac{1}{2} x^2 - \frac{1}{6} i x^3 + - \dots$$

$$= \underbrace{\left(1 - \frac{1}{2} x^2 + - \dots\right)}_{\cos x} + i \underbrace{\left(x - \frac{1}{6} x^3 + - \dots\right)}_{\sin x}$$

$$\underline{e^{ix} = \cos x + i \sin x}$$