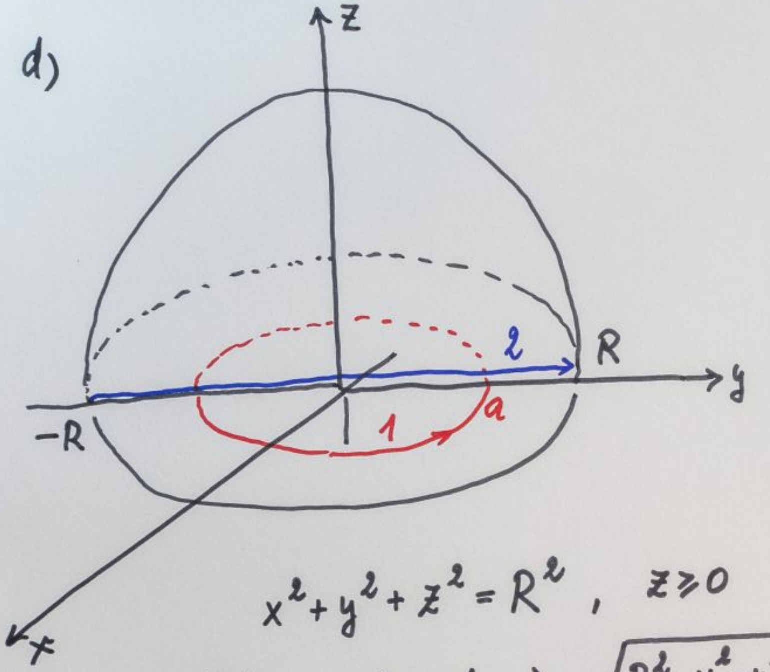
d)



$$x^2 + y^2 + z^2 = R^2 \ , \quad z \geq 0$$

Höhe: $h = z(x,y) = \sqrt{R^2 - x^2 - y^2}$

Höhenlinien: $h = h_0 = \text{const}$

$$x^2 + y^2 = R^2 - h_0^2 \ , \quad h_0 \leq R$$

Kreise

$$\operatorname{grad} h = \frac{-2x\,\vec{i} - 2y\,\vec{j}}{2\sqrt{R^2 - x^2 - y^2}} = -\frac{x\,\vec{i} + y\,\vec{j}}{\sqrt{R^2 - x^2 - y^2}}$$

Reisegleichung: $\dfrac{dh}{dt} = \vec{V} \cdot \operatorname{grad} h$

1. Weg: Höhenlinie

Parametrisierung: $\vec{r} = a\cos\omega t \cdot \vec{i} + a\sin\omega t \cdot \vec{j}$

$\vec{V} = -a\omega\sin\omega t \cdot \vec{i} + a\omega\cos\omega t \cdot \vec{j}$

$$\frac{dh}{dt} = -\frac{-a\omega\sin\omega t \cdot a\cos\omega t + a\omega\cos\omega t \cdot a\sin\omega t}{\sqrt{R^2 - a^2(\cos^2\omega t + \sin^2\omega t)}} = 0$$

2. Weg:

Parametrisierung: $x = 0, \quad y = v_0 t - R \ , \quad v_0 = \text{const}$

$0 \leq t \leq \dfrac{2R}{v_0}$

$\vec{V} = v_0\,\vec{j}$

$$\frac{dh}{dt} = -\frac{v_0\, y}{\sqrt{R^2 - y^2}} = -\frac{v_0(v_0 t - R)}{\sqrt{R^2 - (v_0^2 t^2 - 2v_0 R t + R^2)}}$$

$$= \frac{v_0(R - v_0 t)}{\sqrt{v_0 t(2R - v_0 t)}}$$

- $t = 0, \ t = \dfrac{2R}{v_0} \ : \ \dfrac{dh}{dt} \to \infty$
- $t = \dfrac{R}{v_0} \ : \ \dfrac{dh}{dt} = 0$
- $t \lessgtr \dfrac{R}{v_0} \ : \ \dfrac{dh}{dt} \gtrless 0$