Differentialrechnung mit einer Variablen

Differentiationsregeln und weitere Ableitungen – Teil 2

c) $y = \dfrac{1}{u(x)}$

$$y' = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon}\left[\frac{1}{u(x+\varepsilon)} - \frac{1}{u(x)}\right] = \lim_{\varepsilon \to 0} \frac{u(x) - u(x+\varepsilon)}{\varepsilon \cdot u(x+\varepsilon) \cdot u(x)}$$

$$= \lim_{\varepsilon \to 0} (-1)\frac{u(x+\varepsilon) - u(x)}{\varepsilon} \cdot \lim_{\varepsilon \to 0} \frac{1}{u(x+\varepsilon) \cdot u(x)}$$

$$= -u' \cdot \frac{1}{u^2} = -\frac{u'}{u^2}$$, Reziprokenregel

Beispiele:

- $y = x^{-n} = \dfrac{1}{x^n}$, n ganz

  $u(x) = x^n$, $u' = n x^{n-1}$

  $$y' = -\frac{n x^{n-1}}{x^{2n}} = -n x^{-n-1}$$

- $y = e^{-x} = \dfrac{1}{e^x}$

  $u(x) = e^x$, $u' = e^x$

  $$y' = -\frac{e^x}{e^{2x}} = -e^{-x}$$

d) $y = \dfrac{u(x)}{v(x)} = u(x) \cdot \dfrac{1}{v(x)}$

$$y' = u' \cdot \frac{1}{v} + u \frac{d}{dx}\left(\frac{1}{v}\right) = \frac{u'}{v} - u\frac{v'}{v^2}$$

Produkt- und Reziprokenregel

$$= \frac{u'v - uv'}{v^2}$$, Quotientenregel

Beispiele:

- $y = \tan x = \dfrac{\sin x}{\cos x}$ $\quad u = \sin x \quad v = \cos x$

  $\quad u' = \cos x \quad v' = -\sin x$

  $$y' = \frac{\cos x \cdot \cos x + \sin x \cdot \sin x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

- $y = \cot x = \dfrac{1}{\tan x}$, $u = \tan x$

  $u' = \dfrac{1}{\cos^2 x}$

  $$y' = -\frac{1}{\cos^2 x \cdot \tan^2 x} = -\frac{1}{\sin^2 x}$$