Komplexe Zahlen

Rechenregeln

gegeben: $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$

a) $z_1 = z_2$: $x_1 = x_2$ <u>und</u> $y_1 = y_2$

b) Addition: $z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$

c) Subtraktion: $z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$

d) Multiplikation: $z_1 \cdot z_2 = (x_1 + iy_1)(x_2 + iy_2)$

$$z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$$

speziell: $z\bar{z} = (x+iy)(x-iy) = \underbrace{x^2 + y^2 \geq 0}_{\text{reell}}$

aber: $z^2 = (x+iy)(x+iy)$
$= (x^2 - y^2) + 2ixy$, komplex

e) Division

Kehrwert

$$\frac{1}{z} = \frac{1}{x+iy} = \frac{x-iy}{(x+iy)(x-iy)} = \frac{x-iy}{x^2+y^2}$$

$$\mathrm{Re}\,\frac{1}{z} = \frac{x}{x^2+y^2} \quad , \quad \mathrm{Im}\,\frac{1}{z} = -\frac{y}{x^2+y^2}$$

Beispiel: $\frac{1}{i} \rightarrow z = i$, $x = 0$, $y = 1$

$$\mathrm{Re}\,\frac{1}{i} = 0 \quad , \quad \mathrm{Im}\,\frac{1}{i} = -\frac{1}{1} = -1$$

$$\left(\frac{1}{i} = -i\right)$$

Quotient

$$\frac{z_1}{z_2} = z_1 \cdot \frac{1}{z_2} = \frac{z_1 \bar{z}_2}{z_2 \bar{z}_2} = \frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} + i\,\frac{x_2 y_1 - x_1 y_2}{x_2^2 + y_2^2}$$