Die Exponentialfunktion

Wachstum und Zerfall

$$y = Ae^{cx} \quad , \quad A = \text{const} = y(0)$$

$$\dim A = \dim y$$

$$c = \text{const} \begin{cases} >0 & \text{exp. Wachstum} \\ <0 & \text{exp. Zerfall} \end{cases}$$

$$\underline{\underline{y'}} = \frac{dy}{dx} = Ace^{cx} = \underline{\underline{cy}}$$

Beispiel: $I(x) = I_0 e^{-\alpha x} \quad , \quad \alpha > 0 \, , \, c = -\alpha < 0$ Zerfall

$$I'(x) = -\alpha I(x)$$

Wachstum: $y = Ae^{ct}$, $c > 0$

Verdopplungszeit $T_{(2)}$

$$y(t_0 + T_{(2)}) \overset{!}{=} 2y(t_0)$$

$$Ae^{c(t_0 + T_{(2)})} = 2Ae^{ct_0}$$

$$e^{cT_{(2)}} = 2$$

$$cT_{(2)} = \ln 2$$

$$T_{(2)} = \frac{1}{c} \ln 2$$

Zerfall: $y = Ae^{-ct}$, $c > 0$

Halbwertszeit $T_{(1/2)}$

$$y(t_0 + T_{(1/2)}) \overset{!}{=} \frac{1}{2} y(t_0)$$

$$Ae^{-c(t_0 + T_{(1/2)})} = \frac{1}{2} Ae^{-ct_0}$$

$$e^{-cT_{(1/2)}} = \frac{1}{2}$$

$$-cT_{(1/2)} = \ln \frac{1}{2} = -\ln 2$$

$$T_{(1/2)} = \frac{1}{c} \ln 2$$

$$y' = cy + d \quad , \quad d = \text{const}$$

$$y' = c\left(y + \frac{d}{c}\right)$$

$$\left(y + \frac{d}{c}\right)' = c\left(y + \frac{d}{c}\right) \quad , \quad y + \frac{d}{c} \equiv z$$

$$z' = cz$$

$$z = Ae^{cx} = y + \frac{d}{c}$$

$$\underline{\underline{y = Ae^{cx} - \frac{d}{c}}}$$

Probe: L.H.S. $y' = Ace^{cx}$

R.H.S. $c\left(Ae^{cx} - \frac{d}{c}\right) + d$

$$= Ace^{cx} - d + d$$

$$= Ace^{cx}$$