6.) Integration über ein zu Null symmetrisches Intervall

$$\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx$$

$$= \int_{a}^{0} f(-x)\,d(-x) + \int_{0}^{a} f(x)\,dx$$

$$= -\int_{0}^{a} f(-x)\,d(-x) + \int_{0}^{a} f(x)\,dx \quad , \quad d(-x) = -dx$$

$$= \int_{0}^{a} \left[f(-x) + f(x) \right] dx$$

a) ungerade Funktion: $f(-x) = -f(x)$

$$\int_{-a}^{a} f(x)\,dx = 0$$

b) gerade Funktion: $f(-x) = f(x)$

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$