Eigenschaften bestimmter Integrale

1.) $\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$, Zahl

x: „stumme Variable"

2.) Eindeutigkeit

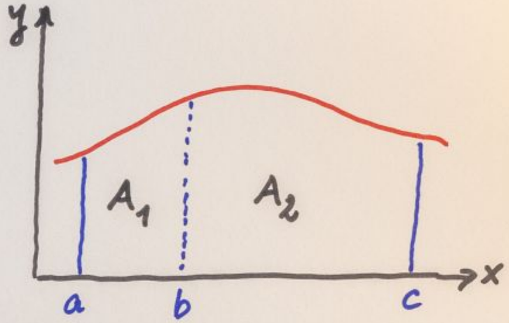
$$F_1'(x) = F_2'(x) = f(x) \;\longrightarrow\; F_1 = F_2 + C$$

$$F_1(b) - F_1(a) = [F_2(b) + C] - [F_2(a) + C]$$
$$= F_2(b) - F_2(a)$$

3.) Linearität

$$\int_a^b [\alpha f(x) + \beta g(x)]\,dx = [\alpha F(x) + \beta G(x)]\Big|_a^b$$
$$= [\alpha F(b) + \beta G(b)] - [\alpha F(a) + \beta G(a)]$$
$$= \alpha [F(b) - F(a)] + \beta [G(b) - G(a)]$$
$$= \alpha \int_a^b f(x)\,dx + \beta \int_a^b g(x)\,dx$$

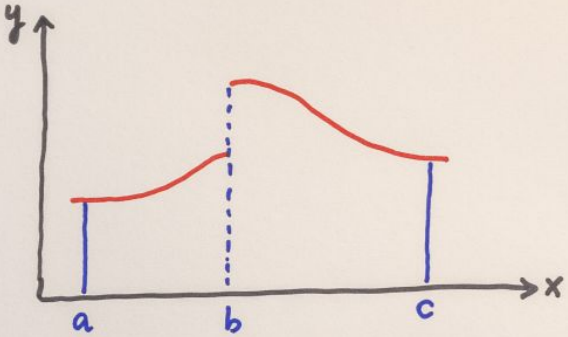
4.) Additivität der Integrationsgrenzen



$a < b < c$

$$A_1 + A_2 = \int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

$$[F(b) - F(a)] + [F(c) - F(b)] = F(c) - F(a)$$



weniger restriktiv als Differentiation

5.) Umkehrung des Integrationsweges

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx \;,\quad F(b) - F(a) = -[F(a) - F(b)]$$

speziell: $\int_a^a f(x)\,dx = 0$