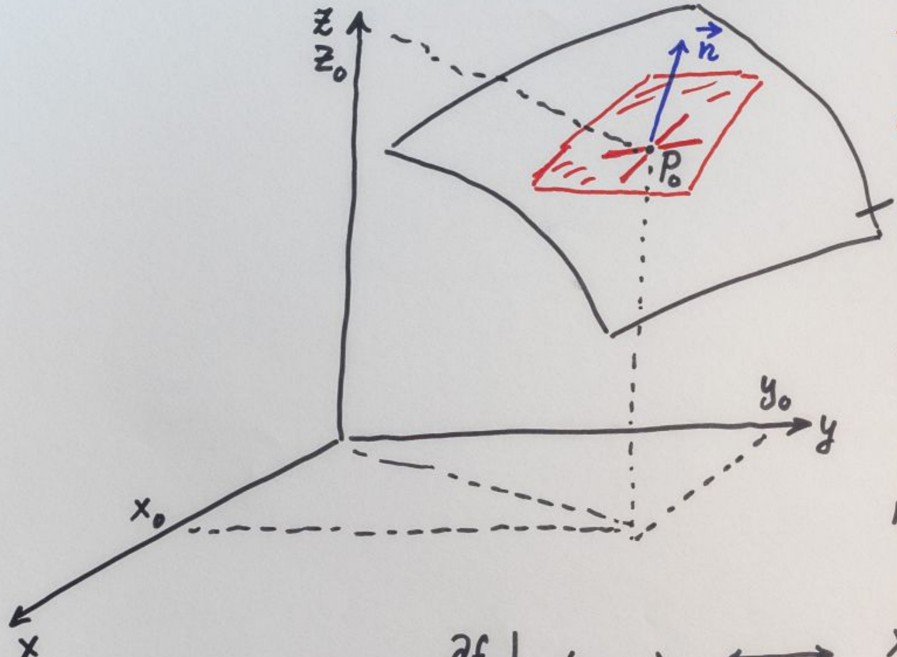
Geometrische Interpretation der partiellen Ableitungen.

Tangentialebene und Normalenvektor einer Fläche



Tangentialebene:

$$(\vec{r} - \vec{r}_0) \cdot \vec{n} = 0$$

$$n_1(x-x_0) + n_2(y-y_0) + n_3(z-z_0) = 0$$

$x = x_0$: $\quad z = z_0 + \left.\frac{\partial f}{\partial y}\right|_{P_0} \cdot (y - y_0) \quad \longleftrightarrow \quad x = x_0: \; n_2(y-y_0) + n_3(z-z_0) = 0 \; \longrightarrow \; n_2 = -\left.\frac{\partial f}{\partial y}\right|_{P_0}, \; n_3 = 1$

$y = y_0$: $\quad z = z_0 + \left.\frac{\partial f}{\partial x}\right|_{P_0} \cdot (x - x_0) \quad \longleftrightarrow \quad y = y_0: \; n_1(x-x_0) + n_3(z-z_0) = 0 \; \longrightarrow \; n_1 = -\left.\frac{\partial f}{\partial x}\right|_{P_0}, \; n_3 = 1$

Normalenvektor:

$$\vec{n} = -\left.\frac{\partial f}{\partial x}\right|_{P_0} \cdot \vec{i} - \left.\frac{\partial f}{\partial y}\right|_{P_0} \cdot \vec{j} + \vec{k}$$

partielle Ableitungen: Komponenten des Normalenvektors in P_0

Tangentialebene: $-\left.\frac{\partial f}{\partial x}\right|_{P_0} \cdot (x-x_0) - \left.\frac{\partial f}{\partial y}\right|_{P_0} \cdot (y-y_0) + (z-z_0) = 0$

$$\left(-\left.\frac{\partial f}{\partial x}\right|_{P_0} \cdot \vec{i} - \left.\frac{\partial f}{\partial y}\right|_{P_0} \cdot \vec{j} + \vec{k}\right) \cdot \left((x-x_0)\,\vec{i} + (y-y_0)\,\vec{j} + (z-z_0)\,\vec{k}\right) = 0$$