Beispiele für die Berechnung partieller Ableitungen

Keine neuen Rechenregeln!

a) $z = f(x,y) = x^3 y + x y^2 + x + y^2 + 1$

$$\frac{\partial z}{\partial x} = 3x^2 y + y^2 + 1 \quad , \quad \frac{\partial z}{\partial y} = x^3 + 2xy + 2y$$

b) $z = f(x,y) = \frac{x}{x+y}$ (Quotientenregel)

$$\frac{\partial z}{\partial x} = \frac{(x+y) - x}{(x+y)^2} = \frac{y}{(x+y)^2}$$

$$\frac{\partial z}{\partial y} = \frac{-x}{(x+y)^2}$$

Zweite Ableitungen

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \equiv \frac{\partial^2 z}{\partial x^2} \equiv z_{xx} \quad , \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \equiv \frac{\partial^2 z}{\partial y^2} \equiv z_{yy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \equiv \frac{\partial^2 z}{\partial y\,\partial x} \equiv z_{xy} \quad , \quad \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \equiv \frac{\partial^2 z}{\partial x\,\partial y} \equiv z_{yx}$$

gemischte 2. Ableitung

$$\frac{\partial^2 z}{\partial x\,\partial y} = \frac{\partial^2 z}{\partial y\,\partial x} \quad , \text{ Schwarz}$$

Vertauschbarkeit der gemischten 2. Ableitungen

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zu b)

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{(x+y)^2 - y \cdot 2(x+y)}{(x+y)^4}$$

$$= \frac{x^2 + \cancel{2xy} + y^2 - \cancel{2xy} - 2y^2}{(x+y)^4} = \frac{x^2 - y^2}{(x+y)^4}$$

$$= \underline{\underline{\frac{x-y}{(x+y)^3}}}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{-(x+y)^2 + x \cdot 2(x+y)}{(x+y)^4}$$

$$= \frac{-x^2 - \cancel{2xy} - y^2 + 2x^2 + \cancel{2xy}}{(x+y)^4}$$

$$= \frac{x^2 - y^2}{(x+y)^4} = \underline{\underline{\frac{x-y}{(x+y)^3}}}$$