Differentialrechnung mit einer Variablen

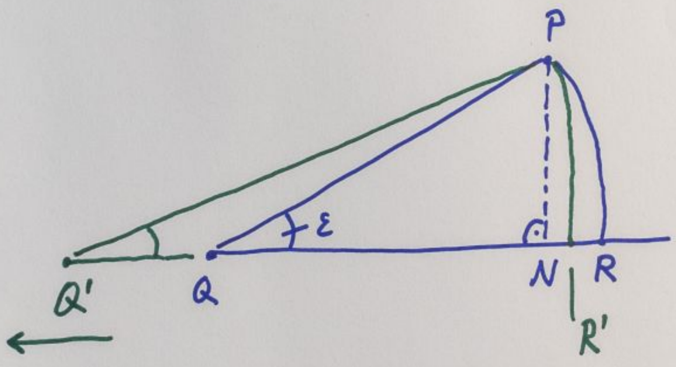
Grenzwerte und Ableitungen – Teil 2

f) $y = \sin x$

$$y' = \lim_{\varepsilon\to 0} \frac{\sin(x+\varepsilon) - \sin x}{\varepsilon} = \lim_{\varepsilon\to 0} \frac{(\sin x \cos\varepsilon + \cos x \sin\varepsilon) - \sin x}{\varepsilon}$$ (Additionstheoreme)

$$= \sin x \cdot \lim_{\varepsilon\to 0} \frac{\cos\varepsilon - 1}{\varepsilon} + \cos x \cdot \lim_{\varepsilon\to 0} \frac{\sin\varepsilon}{\varepsilon} \longrightarrow y' = \cos x$$

$$\lim_{\varepsilon\to 0} \frac{\sin\varepsilon}{\varepsilon} \; ?$$



$$\left.\begin{aligned} \sin\varepsilon &= \frac{|\overline{PN}|}{|\overline{PQ}|} \\ \widehat{PR} &= \varepsilon \cdot |\overline{PQ}| \end{aligned}\right\} \quad \frac{\sin\varepsilon}{\varepsilon} = \frac{|\overline{PN}|}{\cancel{|\overline{PQ}|}} \cdot \frac{\cancel{|\overline{PQ}|}}{\widehat{PR}} = \frac{|\overline{PN}|}{\widehat{PR}}$$

$$\lim_{\varepsilon\to 0} \frac{|\overline{PN}|}{\widehat{PR}} = 1$$

$$\underline{\underline{\lim_{\varepsilon\to 0} \frac{\sin\varepsilon}{\varepsilon} = 1}}$$

$$\lim_{\varepsilon\to 0} \frac{\cos\varepsilon - 1}{\varepsilon} = \lim_{\varepsilon\to 0} \left(\frac{\cos\varepsilon - 1}{\varepsilon} \cdot \frac{\cos\varepsilon + 1}{\cos\varepsilon + 1}\right) = \lim_{\varepsilon\to 0} \frac{\cos^2\varepsilon - 1}{\varepsilon(\cos\varepsilon + 1)} = \lim_{\varepsilon\to 0} \frac{-\sin^2\varepsilon}{\varepsilon(\cos\varepsilon + 1)}$$

$$= -\underbrace{\lim_{\varepsilon\to 0} \frac{\sin\varepsilon}{\varepsilon}}_{1} \cdot \underbrace{\lim_{\varepsilon\to 0} \frac{\sin\varepsilon}{\cos\varepsilon + 1}}_{0} = 0$$

g) $y = \cos x$

$$y' = -\sin x$$