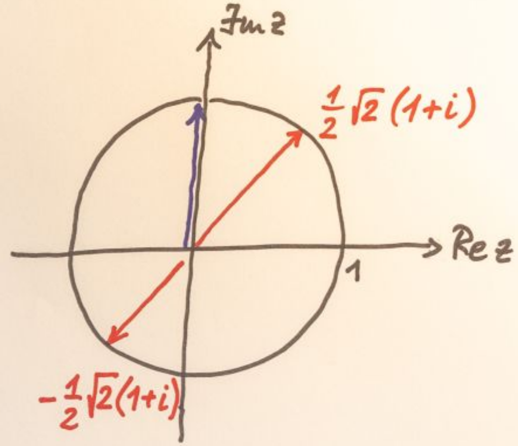
Komplexe Zahlen

Kreisteilung: Beispiele

2) $\sqrt{i}$: $n=2$, $k=0,1$

$r=1$, $\varphi=\frac{\pi}{2}$

$$\sqrt{i} = \cos\left(\frac{\pi}{4}+\pi k\right) + i\sin\left(\frac{\pi}{4}+\pi k\right) \quad \begin{cases} k=0 & \cos\frac{\pi}{4}+i\sin\frac{\pi}{4} = \frac{1}{2}\sqrt{2}\,(1+i) \\ k=1 & \cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4} = -\frac{1}{2}\sqrt{2}\,(1+i) \end{cases}$$



3) n-te Einheitswurzeln

$\sqrt[n]{1}$: n, $k=0,1,\dots,(n-1)$

$r=1$, $\varphi=0$

$$\sqrt[n]{1} = \cos\left(2\pi\frac{k}{n}\right) + i\sin\left(2\pi\frac{k}{n}\right)$$

reell. $k=0$: $z_0=1$

$2\pi\frac{k}{n} \overset{!}{=} \pi \rightarrow n=2k$ gerade : $z_{n/2}=-1$

komplex: $j<\frac{n}{2}$ ganz : $z_j = \cos\left(2\pi\frac{j}{n}\right) + i\sin\left(2\pi\frac{j}{n}\right)$

$$z_{n-j} = \cos\frac{2\pi(n-j)}{n} + i\sin\frac{2\pi(n-j)}{n}$$

$$= \underbrace{\cos 2\pi}_{1}\cos\frac{2\pi j}{n} + \cancel{\sin 2\pi \sin}\frac{2\pi j}{n} + i\left[\cancel{\sin 2\pi\cos\frac{2\pi j}{n}} - \underbrace{\cos 2\pi}_{1}\sin\frac{2\pi j}{n}\right]$$

(Additionstheoreme)

$$z_{n-j} = \cos\frac{2\pi j}{n} - i\sin\frac{2\pi j}{n} = \bar{z}_j$$

Kreisradius $\sqrt[n]{r}$

n Punkte